# Reliability Assessment of Distribution Networks

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# Basic definition: *Outages*

- Outage is the non-availability of operation of certain equipment
- Outage may or may not cause interruption.
- Outage can be of:
  - ***Forced outage***: Equipment taken out-of-service on emergency basis
  - ***Scheduled outage***: Equipment taken out-of-service for preventive maintenance
  - ***Partial outage***: Equipment operational with reduced capacity

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# Basic definition: *Interruption*

- Interruption is the loss of service to one or more consumers. An interruption may occur due to one or more component outages.
- Interruption can be of the following types:
  - *Scheduled interruption*: Due to scheduled outages
  - *Momentary interruption*: Duration few seconds
  - *Temporary interruption*: Duration few minutes
  - *Sustained interruption*: Duration few hours

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# Preparation of outage reports

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According to an IEEE committee report, the following basic information should be included in an equipment outage report:

- Type, design, manufacturer, and other descriptions for classification purposes
- Date of installation, location on system, length in the case of a line
- Mode of failure (short-circuit, false operation, etc.)
- Cause of failure (lightning, tree, etc.)
- Times (both out of service and back in service, rather than outage duration alone), date, meteorological conditions when the failure occurred
- Type of outage, forced or scheduled, transient or permanent

# Distribution system reliability

The major challenges of reliability assessment:

- Insufficient data on reliability performance
- Most of the interruptions of distribution networks are caused by severe weather-related interruptions and for inadequate maintenance.
- Distribution system reliability can be improved by the timely identification and response to failures.

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# Reliability Indices

- There are various types of reliability indices. They are:
  - Indices for sustained interruptions
  - Indices for momentary interruptions
  - Load and energy-based indices

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# Usage of the reliability indices

- Most utilities use one or more of the reliability indices to understand how the distribution system is performing.

- For example, some of the reliability indices which are formulated for sustained interruptions may not reveal momentary interruptions.

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# Benefits of reliability-based study

- A *reliability assessment study* can help to quantify the impact of design improvement options.
- Typical improvement options that can be studied based on a *predictive reliability model* include the following:
  - New feeders and feeder expansions
  - Load transfers between feeders
  - New substation and substation expansions
  - New feeder tie points
  - Locations for sectionalizing switches
  - Feeder automation
  - Replacement of aging equipment

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# Costs associated with system reliability

- Investment cost (utility side) to achieve a desired level of reliability
- Customer interruption cost which includes the cost of production loss for process industries
- Customer damage cost (utility side), if applicable

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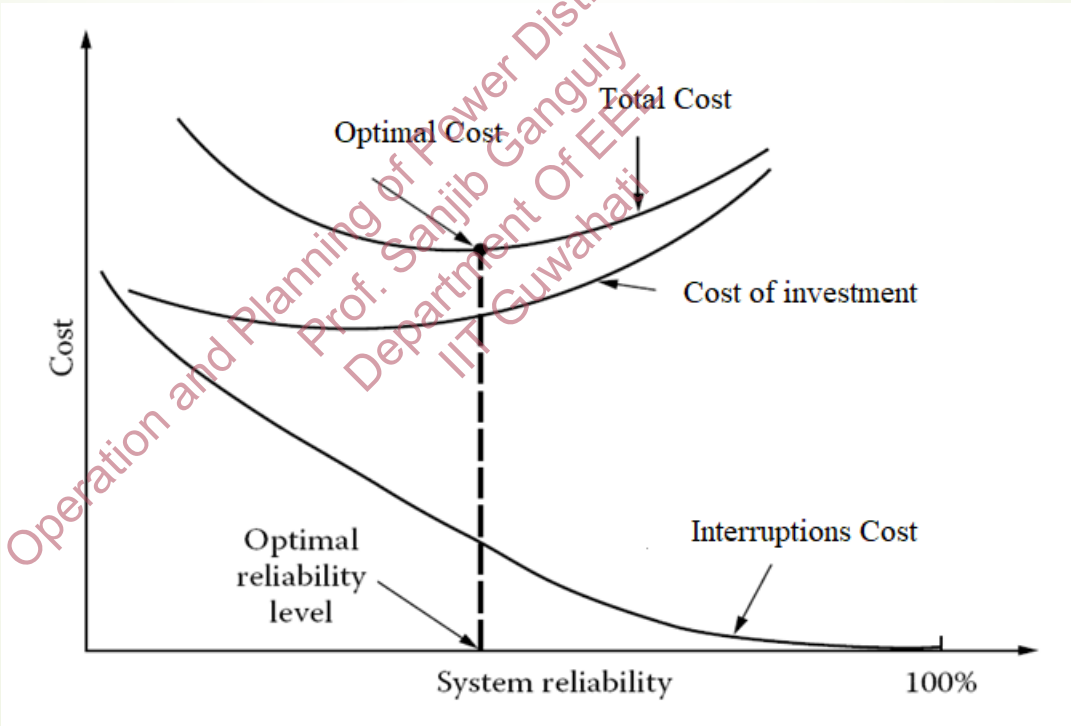
# Reliability and Cost: Conflicts each other

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# Reliability index: SAIFI

- *System average interruption frequency index (SAIFI)* (*sustained interruptions*): This index is designed to provide the information about the average frequency of sustained interruptions per customer over a predefined service area.

$$SAIFI = \frac{total\ number\ of\ customer\ interruptions}{total\ number\ of\ customers\ served} \qquad SAIFI = \frac{\sum N_i}{N_T}$$

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Where
$N_i$ is the number of interrupted customers for each interruption event during reporting period
$N_T$ is the total number of customers served for the area being indexed

# Reliability index: SAIDI

- *System average interruption duration index (SAIDI)*: This index is commonly referred to as customer minutes of interruption or customer hours, and is designed to provide information about the average time the customers are interrupted.

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$$SAIDI = \frac{\sum customer\ interruption\ durations}{total\ number\ of\ customers\ served}$$

$$SAIDI = \frac{\sum r_i N_i}{N_T}$$

Where $r_i$ is the restoration time for each interruption event.

# Reliability index: CAIDI

- *Customer average interruption duration index (CAIDI)*: It represents the average time required to restore service to the average customer per sustained interruption.

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$$CAIDI = \frac{\sum customer\ interruption\ durations}{total\ number\ of\ customers\ interruptions}$$

$$CAIDI = \frac{\sum r_i N_i}{\sum N_i} = \frac{SAIDI}{SAIFI}$$

# Reliability index: CTAIDI

- *Customer total average interruption duration index (CTAIDI)*: For customers who actually experienced an interruption, this index represents the total average time in the reporting period they were without service.
- This index is a hybrid of CAIDI and is calculated the same except that customers with multiple interruptions are counted only once.

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$$CTAIDI = \frac{\sum customer\ interruption\ durations}{total\ number\ of\ customers\ interrupted} \qquad CTAIDI = \frac{\sum R_i N_i}{CN}$$

Where, *CN* is total number of customers who have experienced a sustained interruption during the reporting period.

# Reliability indices: CAIFI

- *Customer average interruption frequency index (CAIFI)*: This index gives the average frequency of sustained interruptions for those customers experiencing sustained interruptions.
- The customer is counted only once regardless of the number of times interrupted.

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$$CAIFI = \frac{total\ number\ of\ customer\ interruptions}{total\ number\ of\ customers\ interrupted} \qquad CAIFI = \frac{\sum N_i}{CN}$$

# Reliability index: ASAI

- *Average service availability index (ASAI)*: This index represents the fraction of time (often in percentage) that a customer had power without interruption during 1 year or the defined reporting period.

$$ASAI = \frac{customer\ hours\ service\ availability}{customer\ hours\ service\ demand}$$

$$ASAI = \frac{N_T\left(number\ of\ \frac{hours}{year}\right) - \sum r_i N_i}{N_T(number\ of\ hours/year)}$$

$\sum r_i N_i$ represents total duration of interruption in a year

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# Reliability index: ASIFI

- *Average system interruption frequency index (ASIFI)*: This index is specifically designed to calculate reliability based on load rather than number of customers.
- It is an important index for areas that serve industrial/commercial customers.
- It is also used by utilities that do not have precise customer tracking systems.

$$ASIFI = \frac{connected\ kVA\ interrupted}{total\ connected\ kVA\ served}$$

$$ASIFI = \frac{\sum L_i}{L_T}$$

Where
$\sum L_i$ is the total connected kVA load interrupted for each interruption event
$L_T$ is the total connected kVA load served

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# Reliability index: ASIDI

- *Average system interruption duration index (ASIDI)*: This index is formulated with the same philosophy as ASIFI, but it provides information on system average duration of interruptions. Thus,

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$$ASIDI = \frac{connected\ kVA\ duration\ interrupted}{total\ connected\ kVA\ served}$$

$$ASIDI = \frac{\sum r_i\ L_i}{L_T}$$

# Reliability index: CEMIn

- *Customers experiencing multiple interruptions (CEMIn)*: This index is designed to track the number (*n*) of sustained interruptions to a specific customer.
- Its purpose is to help identify customer trouble that cannot be seen by using averages.

$$CEMI_n = \frac{total\ number\ of\ customers\ that\ experienced\ more\ sustained\ interruptions}{total\ number\ of\ customers\ served}$$

$$CEMI_n = \frac{CN_{(k>n)}}{N_T}$$

where $CN_{(k>n)}$ is the total number of customers who have experienced more than *n* sustained interruptions during the reporting period.

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# Reliability index: MAIFI

- *Momentary average interruption frequency index (MAIFI)*: *This index is very similar to* SAIFI, but it tracks the average frequency of momentary interruptions.

$$MAIFI = \frac{total\ number\ of\ customer\ momentary\ interruptions}{total\ number\ of\ customers\ served}$$

$$MAIFI = \frac{\sum ID_i\ N_i}{N_T}$$

where $ID_i$ is the number of interrupting device operations.

MAIFI is the same as SAIFI, but it is for short-duration rather than long-duration interruptions.

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# Reliability index: $MAIFI_E$

- *Momentary average interruption event frequency index ($MAIFI_E$)*: This index is very similar to SAIFI, but it tracks the average frequency of momentary interruption events.

$$\mathrm{MAIFI_E} = \frac{total\ number\ of\ customer\ momentary\ interruption\ events}{total\ number\ of\ customers\ served}$$

$$MAIFI_E = \frac{\sum ID_E\ N_i}{N_T}$$

where $ID_E$ is the interrupting device events during reporting period.

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# Reliability index: CEMSMI*n*

- *Customers experiencing multiple sustained interruption and momentary interruption events (CEMSMIn):* This index is formulated for tracking the number (*n*) of both sustained interruption and momentary interruption events to a set of specific customers.

$$CEMSMI_n = \frac{total\ number\ of\ customers\ that\ experienced\ more\ than\ n\ interruptions}{total\ number\ of\ customers\ served}$$

$$CEMSMI_n = \frac{CNT_{(k>n)}}{N_T}$$

Where $CNT_{(k>n)}$ is the total number of customers who have experienced more than n sustained interruption and momentary interruption events during the reporting period.

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# Load and energy based reliability indices

- There are also load- and energy-based indices. In determination of such indices, one has to know the average load at each load bus. This average load $L_{avg}$ at a bus is found from:

$$L_{avg} = L_{peak}\, LF$$

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Where,
$L_{avg}$ is the peak load (demand)
*LF* is the load factor

# Load and energy based reliability index: ENS

- *Energy not supplied index (ENS)*: This index represents the total energy *not supplied* by the system and is expressed as:

$$ENS = \sum L_{avg,i}\, r_i$$

Where $L_{avg,i}$ is the average load connected to load point i

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# Load and energy based reliability index: AENS

- *Average energy not supplied (AENS)*: This index represents the average energy not supplied by the system.

$$AENS = \frac{total\ energy\ not\ supplied}{total\ number\ of\ customers\ served}$$

$$AENS = \frac{\sum L_{avg,\,i}\, r_i}{N_T}$$

This index is the same as the average system curtailment index (ASCI).

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# Load and energy based reliability index: ACCI

- *Average customer curtailment index (ACCI)*: This index represents the total energy not supplied per affected customer by the system.

$$ACCI = \frac{total\ energy\ not\ supplied}{total\ number\ of\ customers\ affected}$$

$$ACCI = \frac{\sum L_{avg\ i}\ r_i}{CN}$$

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It is a useful index for monitoring the changes of average energy *not supplied* between one calendar year and another.

# Example…

| Load Point | Number of customers ($N_i$) | Average Load Connected (kW)($L_{avg,i}$) |
|---|---|---|
| 1 | 250 | 2300 |
| 2 | 300 | 3700 |
| 3 | 200 | 2500 |
| 4 | 250 | 1600 |
| | $N_T$=1,000 | $L_T$=10100 |

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- The information given in the Tables is for a distribution network. Assume that the duration of interruption is the same as the restoration time. Determine the following reliability indices:

a. SAIFI, b. CAIFI, c. SAIDI, d. CAIDI, e. ASAI, f. ASIDI, g. ENS, h. AENS, i. ACCI

Curtsey: T. Gonen. *Electric Power Distribution System Engineering*

# Example…

**Annual Interruption Events**

| Load Point Affected | Number of Customers Interrupted ($N_i$) | Load Interrupted (kW) ($L_i$) |
|---|---|---|
| 1 | 250 | 2300 |
| 2 | 200 | 2500 |
| 3 | 250 | 1600 |
| 4 | 250 | 1600 |
| | 950 | 8000 |

| Load point Affected | Duration of Interruptions (hr) ($d_i=r_i$) | Customer Hours Curtailed ($r_iXN_i$) | Energy Not Supplied (kWh) ($r_iXL_i$) |
|---|---|---|---|
| 1 | 2 | 500 | 4600 |
| 2 | 3 | 600 | 7500 |
| 3 | 1 | 250 | 1600 |
| 4 | 1 | 250 | 1600 |
| | | 1600 | 15300 |

CN, number of customers affected = 250+200+250 = 700

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- The information given in the Tables is for a distribution network. Assume that the duration of interruption is the same as the restoration time. Determine the following reliability indices:

a. SAIFI, b. CAIFI, c. SAIDI, d. CAIDI, e. ASAI, f. ASIDI, g. ENS, h. AENS, i. ACCI

# Solution…

a. SAIFI $= \frac{\sum N_i}{N_T} = \frac{950}{1000} = 0.95$ interruptions/customer served

b. CAIFI $= \frac{\sum N_i}{CN} = \frac{950}{700} = 1.357$ interruptions/customer affected

c. SAIDI $= \frac{\sum r_i N_i}{N_T} = \frac{1600}{1000} = 1.6$ hr/customer served = 96 min/customer served

d. CAIDI $= \frac{\sum r_i N_i}{\sum N_i} = \frac{1600}{950} = 1.684$ hr/customer interrupted = 101.05 min/customer interrupted

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# Solution…

e. ASAI $= \frac{N_T 8760 - \sum r_i N_i}{N_T 8760} = \frac{1000 \times 8760 - 1600}{1000 \times 8760} = 0.999817$ unit hour availability

f. ASIDI $= \frac{\sum r_i L_i}{L_T} = \frac{15300}{10100} = 1.515$

g. ENS$= \sum L_{avg,i} r_i = 15300$ kWh

h. AENS $= \frac{ENS}{N_T} = \frac{15300}{1000} = 15.3$ kWh/customer affected

i. ACCI $= \frac{ENS}{CN} = \frac{15300}{700} = 21.857$ kWh/customer affected

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# Exercise:

## Load data

| Load point | Number of customers | Average load connected (kW) |
|---|---|---|
| 1 | 1800 | 8400 |
| 2 | 1300 | 6000 |
| 3 | 900 | 4600 |

Given that, number of customers interrupted/affected=2200

## Annual interruption event data

| Load point affected | Number of customers interrupted | Load interrupted (kW) | Duration of interruption (hr) |
|---|---|---|---|
| 2 | 800 | 3600 | 3 |
| 3 | 600 | 2800 | 3 |
| 3 | 300 | 1800 | 2 |
| 3 | 600 | 2800 | 1 |
| 2 | 500 | 2400 | 1.5 |
| 3 | 300 | 1800 | 1.5 |

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Determine SAIFI, CAIFI, SAIDI, CAIDI, ASAI, ASIDI, ENS, AENS, ACCI

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# Basic mathematical concepts of reliability: General Reliability function

- The probability of failure of a given component (or system) is defined as a function of time as:

$$P(T \leq t) = F(t) \quad t \geq 0$$

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where,
$T$ is a random variable representing the failure time
$F(t)$ is the probability that component will fail by time $t$

# Basic mathematical concepts of reliability: General Reliability function

- Here, $F(t)$ is the failure distribution function, which can be called as the *unreliability function*. Therefore, the probability that the component will not fail at a given time $t$ is defined as the *reliability of the component*. Thus, the *reliability function* can be expressed as:

$$R(t) = 1 - F(t)$$

where,

$R(t)$ is the reliability function

$F(t)$ is the unreliability function

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# Basic mathematical concepts of reliability: General Reliability function

- If the time-to-failure random variable *T* has a density function $f(t)$,

$$R(t) = 1 - F(t)$$

$$= 1 - \int_0^t f(t)dt \quad = \int_t^\infty f(t)dt$$

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# Basic mathematical concepts of reliability: General Reliability function

- The probability of failure of a given system in a particular time interval ($t_1$, $t_2$) can be written in terms of the unreliability function, as:

$$\int_{t_1}^{t_2} f(t)dt = \int_{0}^{t_2} f(t)dt - \int_{0}^{t_1} f(t)dt = F(t_2) - F(t_1)$$

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# Basic mathematical concepts of reliability: General Reliability function

- The probability of failure of a given system in a particular time interval ($t1$, $t2$) can be written in terms of the reliability function, as:

$$\int_{t_1}^{t_2} f(t)dt = \int_{t_1}^{\infty} f(t)dt - \int_{t_2}^{\infty} f(t)dt = R(t_1) - R(t_2)$$

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# Basic mathematical concepts of reliability: General Reliability function

- The rate of failures in a given time interval ($t_1$, $t_2$) is defined as the *hazard rate*, or *failure rate*, during that interval. It is the probability that a failure per unit time happens in the interval, provided that a failure has not happened before the time $t_1$,*i.e.*, at the beginning of the time interval. Therefore,

$$h(t) = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)}$$

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# Basic mathematical concepts of reliability: General Reliability function

Let, the time interval be defined as: $t_1 = t,\ t_2 = t + \Delta t$

$$h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t . R(t)}$$

$$= \frac{1}{R(t)} \left[ \frac{-dR(t)}{dt} \right]$$

$$= \frac{f(t)}{R(t)}$$

Where, $f(t)$ is the probability density function $-\frac{dR(t)}{dt}$

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# Basic mathematical concepts of reliability: General Reliability function

- By substituting R(t) by F(t), we get,

$$h(t) = \frac{f(t)}{1 - F(t)} \quad \text{Hence,} \quad h(t)dt = \frac{dF(t)}{1 - F(t)}$$

Hence,

$$\int_0^t h(t)dt = -\ln[1 - F(t)] \Big|_0^t$$

$$1 - F(t) = exp\left[-\int_0^t h(t)dt\right]$$

Taking derivatives of this Equation, we get,

$$f(t) = h(t)exp\left[-\int_0^t h(t)dt\right]$$

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# Basic mathematical concepts of reliability: General Reliability function

- Thus, the generalized expression for the reliability function can be written as,

$$R(t) = exp\left[-\int_0^t h(t)dt\right]$$

$h(t) = \lambda$ failures/unit time

- Assume that the hazard or failure function is independent of time, i.e.,

We get the failure density function as, $f(t) = \lambda e^{-\lambda t}$

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# Basic mathematical concepts of reliability: basic single-component concepts

- Theoretically, the *expected life*, that is, the expected time during which a component will survive and perform successfully, can be expressed as:

$$E(T) = \int_0^\infty t f(t) dt$$

$$E(T) = -\int_0^\infty t \frac{dR(t)}{dt} dt$$

$$E(T) = -tR(t)\Big|_0^\infty + \int_0^\infty R(t) dt$$

since, $R_{(t=0)}=1$ $R_{(t=\infty)}=0$

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# Basic mathematical concepts of reliability: basic single-component concepts

- Therefore, the *expected life* can be expressed as:

$$E(T) = \int_0^\infty R(t)dt \quad \text{Or,} \quad E(T) = \int_0^\infty \left\{ exp\left[ -\int_0^t \lambda(t)dt \right] \right\} dt$$

When there is a constant failure rate,

$$E(T) = \int_0^\infty e^{-\lambda t} dt = \frac{1}{\lambda}$$

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# Basic mathematical concepts of reliability: MTTF and MTBF

- If the system is simply replaced by a good system (i.e., no maintenance required) the $E(T)$ useful life is also defined as the *mean time to failure* (MTTF) and denoted as:

$$MTTF = \overline{m} = \frac{1}{\lambda}$$

- If the system has undergone maintenance and repairs the useful life, $E(T)$ is also defined as the *mean time between failures* (MTBF) and denoted as:

$$MTBF = \overline{T} = \overline{m} + \overline{r}$$

Where
$\overline{T}$ is the mean cycle time
$\overline{m}$ is the mean time to failure
$\overline{r}$ is the mean time to repair

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# Basic mathematical concepts of reliability: MTTR

- the *mean time to repair* (MTTR) is defined as the *reciprocal of the average* (or mean) *repair rate* and denoted as:

$$MTTR = \bar{r} = \frac{1}{\mu}$$

where, $\mu$ is the mean repair rate.

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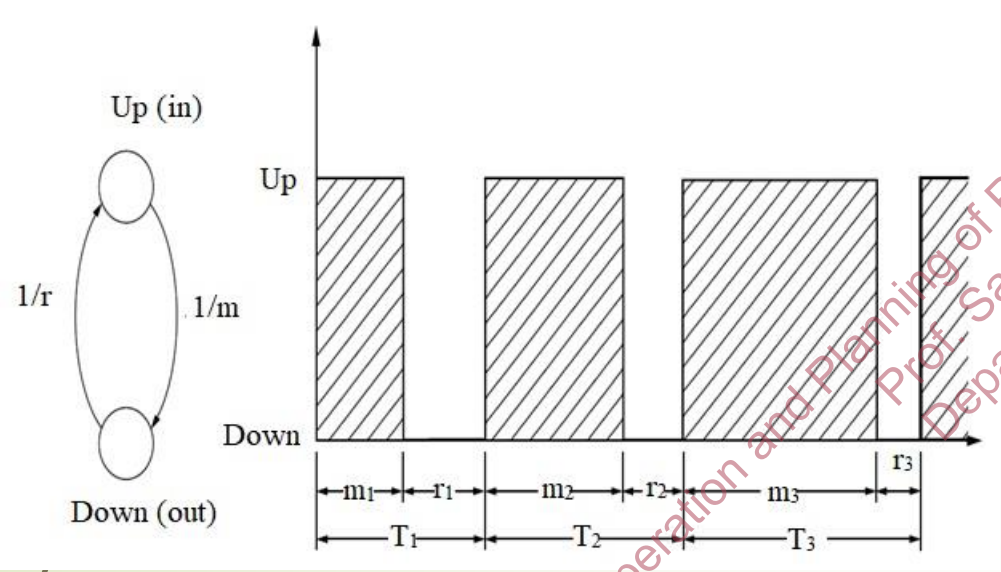
# Computation of MTTR and MTTF in a two-state model

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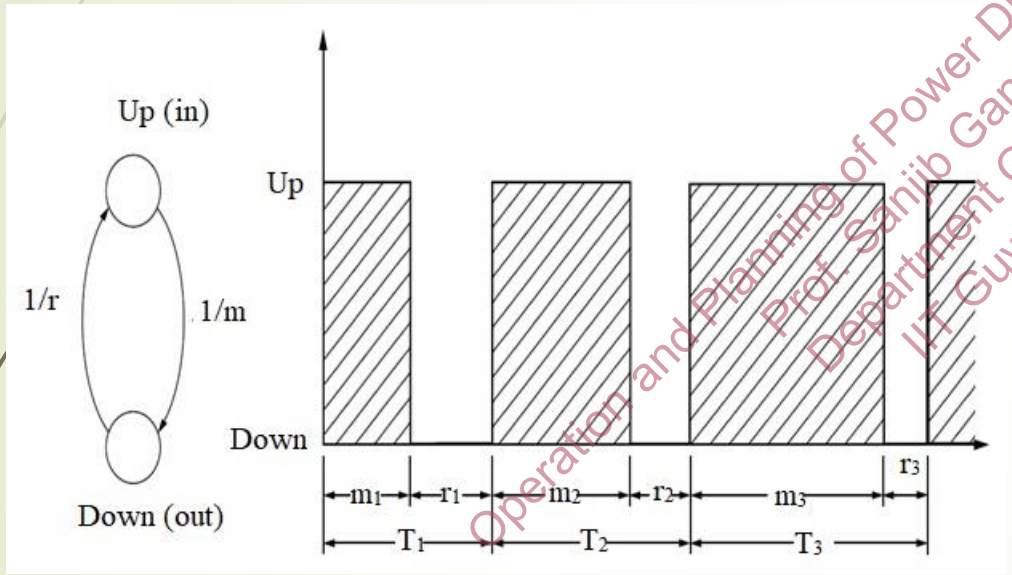
- Consider a two-state model shown in Figure. Assume that the system is either in the *up* (or in) state or in the *down* (or out) state at a given time, as shown in Figure. Therefore, the *mean time to failure* (MTTF) can be reasonably estimated as:

$$\text{MTTF} = \overline{m} = \frac{\sum_{i=1}^{n} m_i}{n}$$

Where,
$\overline{m}$ is the mean time to failure
$m_i$ is the observed time to failure for $i^{th}$ cycle
n is the total number of cycles

# Computation of MTTR and MTTF in a two-state model

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- Similarly, the *mean time to repair* can be reasonably estimated as:

$$\text{MTTR} = \overline{r} = \frac{\sum_{i=1}^{n} r_i}{n}$$

where,
$\overline{r}$ is the mean time to repair
$r_i$ is the time to repair for $i^{th}$ cycle
$n$ is the total number of cycles

Thus, we can write: MTBF=MTTF+MTTR

# Mean cycle time

- The *mean cycle time* defines the average time that it takes for the component to complete one cycle of operation, that is, failure, repair, and restart.

$$\overline{T} = \overline{m} + \overline{r}$$

$$\overline{T} = \frac{1}{\lambda} + \frac{1}{\mu} \quad \text{or,} \quad \overline{T} = \frac{\lambda + \mu}{\lambda\mu}$$

- The reciprocal of the *mean cycle time* is defined as the *mean failure frequency* and denoted as:

$$\overline{f} = \frac{1}{\overline{T}} \qquad \overline{f} = \frac{\lambda\mu}{\lambda + \mu}$$

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# Availability and Unavailability

- In two-state model, it can be assumed that the component is either *up* (i.e., healthy condition) or *down* (i.e., unavailable for service). Therefore, it can be shown that:

$$A + U = 1$$

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where,
A is the availability of component, i.e., the fraction of time component is up/healthy
$A = \overline{U}$ is the unavailability of component, i.e., the fraction of time component is down

# Availability and Unavailability

- Therefore, on the average, as time *t* goes to infinity, it can be shown that the *availability* is:

$$A \triangleq \frac{\overline{m}}{\overline{T}} = \frac{MTTF}{MTBF}$$

or

$$A \triangleq \frac{\overline{m}}{\overline{m} + \overline{r}} = \frac{MTTF}{MTTF + MTTR}$$

- For constant failure rate and repair rate,

$$A = \frac{\mu}{\lambda + \mu}$$

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# Availability and Unavailability

- The *unavailability* can be expressed as:

$$U \triangleq 1 - A$$

$$U = \frac{\overline{r}}{\overline{r} + \overline{m}} = \frac{MTTR}{MTTF + MTTR}$$

- For constant failure rate and repair rate,

$$U = \frac{\lambda}{\lambda + \mu}$$

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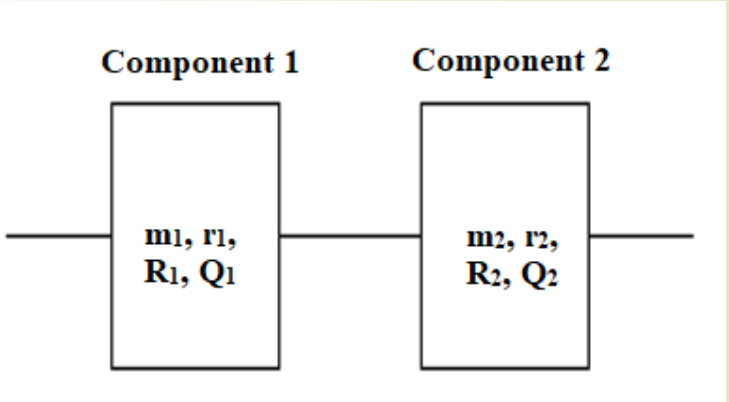
# Reliability of Unrepairable components in series

- Two independent components are connected in series. Therefore, the whole system will successfully operate (or remains healthy) if both components remain healthy, i.e., without a failure.

$$R_{sys}=P\ [E_1 \cap E_2]$$

$$R_{sys}=P\ (E_1)P(E_2)]$$

$$R_{sys}=\prod_{i=1}^{2} R_i$$




Where
$E_i$ is the event that component $i$ (or subsystem $i$) operates successfully
$R_i = P(E_i)$ is the reliability of component $i$ (or subsystem $i$)
$R_{sys}$ is the reliability of system (or system reliability index)

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# Reliability of Unrepairable components in series

- For a series system with *n* independent components, the generalized expression for the system reliability can be written as:

$$R_{sys} = \prod_{i=1}^{n} R_i$$

- Note that, this is the *product rule* or the *chain rule of reliability*. *System reliability will always be less than or equal to the least-reliable component*, i.e.,

$$R_{sys_i} \leq \min\{R_i\}$$

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# Reliability of repairable components in series

- If two independent and repairable components are in series, the *availability* or the *steady-state probability of success* (i.e., operation) of the system can be expressed as:

$$A_{sys} = A_1 \, A_2$$

where,
$A_{sys}$ is the availability of system
$A_1$ is the availability of component 1
$A_2$ is the availability of component 2

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# Reliability of repairable components in series

- Since we know that, $A_1 = \frac{\overline{m_1}}{\overline{r_1} + \overline{m_1}}$ and $A_2 = \frac{\overline{m_2}}{\overline{r_2} + \overline{m_2}}$

- We can write, $A_{sys} = A_1 A_2 = \left(\frac{\overline{m_1}}{\overline{r_1} + \overline{m_1}}\right)\left(\frac{\overline{m_2}}{\overline{r_2} + \overline{m_2}}\right)$

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Where
$\overline{m_1}$ is the mean time to failure of component 1
$\overline{m_2}$ is the mean time to failure of component 2
$\overline{m_{sys}}$ is the mean time to failure of system
$\overline{r_1}$ is the mean time to repair of component 1
$\overline{r_2}$ is the mean time to repair of component 2
$\overline{r_{sys}}$ is the mean time to repair of system

# Reliability of repairable components in series

- The *average frequency of the system failure* is the sum of the average frequency of component-1 failing, given that component 2 is healthy, plus the average frequency of component 2 failing while component 1 is healthy. Thus,

$$\overline{f_{sys}} = A_2 \overline{f_1} + A_1 \overline{f_2}$$

where, $\overline{f_{sys}}$ is the average frequency of system failure

$\overline{f_i}$ is the average frequency of failure of component $i$

$A_i$ is the availability of component $i$

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# Reliability of repairable components in series

- Since we know that, $\overline{f_i} = \frac{1}{\overline{m_i} + \overline{r_i}}$ and $A_i = \frac{\overline{m_i}}{\overline{r_i} + \overline{m_i}}$

- We can write, $\overline{f_{sys}} = \frac{1}{\overline{m_1}+\overline{r_1}} \times \frac{\overline{m_2}}{\overline{m_2}+\overline{r_2}} + \frac{1}{\overline{m_2}+\overline{r_2}} \times \frac{\overline{m_1}}{\overline{m_1}+\overline{r_1}}$

$$A_{sys} = \overline{m_{sys}}\ \overline{f_{sys}}$$

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- Thus, the *mean time to failure* for a given series system with two components can be expressed as

$$\overline{m_{sys}} = \frac{1}{1/\overline{m_1} + 1/\overline{m_2}}$$

# Reliability of repairable components in series

- Hence, the *mean time to failure* of a given series system with *n* components can be expressed as:

$$\overline{m_{sys}} = \frac{1}{1/\overline{m_1} + 1/\overline{m_2} + \cdots + 1/\overline{m_n}}$$

- Hence, the *mean time to failure* of a given series system with *n* components can be expressed as:

$$\lambda_{sys} = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_n$$

- Similarly, the *mean time to repair* for an *n*-component series system is:

$$\overline{r_{sys}} = \frac{\lambda_1 \overline{r_1} + \lambda_2 \overline{r_2} + \lambda_3 \overline{r_3} + \cdots + \lambda_n \overline{r_n}}{\lambda_{sys}}$$

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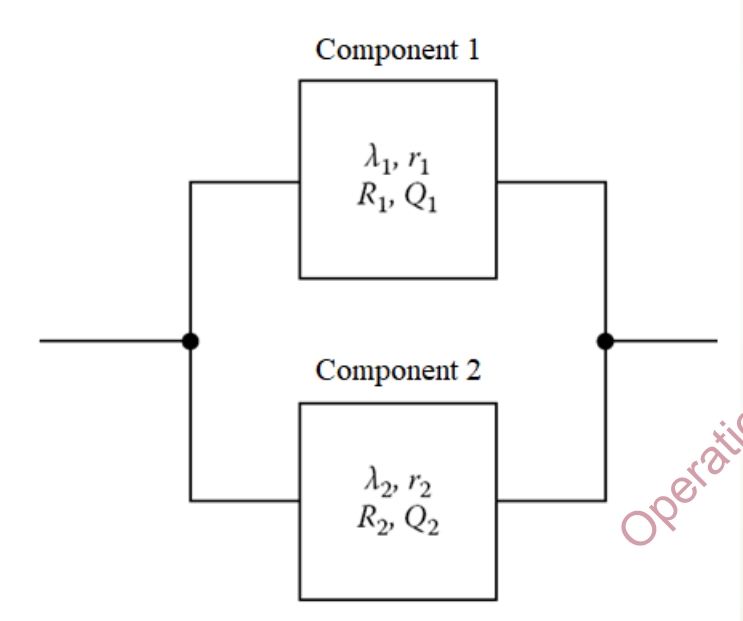
# Reliability of unrepairable components in parallel

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- Figure shows a block diagram for a system that has two independent components connected in parallel.
- Therefore, to have the system fail and not be able to perform its desired function, both components must fail simultaneously.
- Thus, the *system unreliability* is

$$Q_{sys} = P[\overline{E_1} \cap \overline{E_2}] \qquad Q_{sys} = P(\overline{E_1})P(\overline{E_2})]$$

$$Q_{sys} = \prod_{i=1}^{2}(1 - R_i)$$

Where
$\overline{E_i}$ is the event that component *i* fails
$Q_i = P(\overline{E_i})$ is the unreliability of component *i*
$Q_{sys}$ is the unreliability of system (or system unreliability index)

# Reliability of unrepairable components in parallel

- For *m independent* components, the system reliability expression is:

$$R_{sys} = 1 - Q_{sys}$$

$$= 1 - [Q_1 Q_2 Q_3 \ldots Q_m] = 1 - \prod_{i=1}^{m} Q_i$$

$$= 1 - [(1 - R_1)(1 - R_2)\ (1 - R_3)\ldots(1 - R_m)]$$

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# Reliability of repairable components in parallel

- Consider a parallel system with two independent and repairable components.
- Therefore, the *unavailability* or the *steady-state probability of failure of the system* can be expressed as:

$$U_{sys} = U_1 U_2$$

where, $U_1 = 1 - A_1 = \frac{\lambda_1 \overline{r_1}}{1+\lambda_1 \overline{r_1}}$ $\qquad$ $U_2 = 1 - A_2 = \frac{\lambda_2 \overline{r_2}}{1+\lambda_2 \overline{r_2}}$

$$U_{sys} = \left(\frac{\lambda_1 \overline{r_1}}{1+\lambda_1 \overline{r_1}}\right)\left(\frac{\lambda_2 \overline{r_2}}{1+\lambda_2 \overline{r_2}}\right)$$

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# Reliability of repairable components in parallel

- However, the *average frequency of the system failure* is:

$$\overline{f_{sys}} = U_2\overline{f_1} + U_1\overline{f_2}$$

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where

$\overline{f_{sys}}$ is the average frequency of system failure

$\overline{f_i}$ is the average frequency of failure of component *I*

$U_i$ is the unavailability of component *i*

# Reliability of repairable components in parallel

- Since, $\overline{f_1} = \dfrac{\lambda_1}{1 + \lambda_1 \overline{r_1}}$ $\overline{f_2} = \dfrac{\lambda_2}{1 + \lambda_2 \overline{r_2}}$

- We get the following expression:

$$\overline{f_{sys}} = \frac{\lambda_1 \lambda_2 (\overline{r_1} + \overline{r_2})}{(1 + \lambda_1 \overline{r_1})(1 + \lambda_2 \overline{r_2})}$$

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❑ The system *unavailability* can be expressed as:

$$U_{sys} \triangleq \frac{\overline{r_{sys}}}{\overline{T_{sys}}} \qquad U_{sys} = \overline{r_{sys}}\ \overline{f_{sys}}$$

# Reliability of repairable components in parallel

- The *average repair time* (or *downtime*) of the two-component parallel system can be expressed as:

$$\overline{r_{sys}} = \frac{\overline{r_1 r_2}}{\overline{r_1} + \overline{r_2}}$$

- The system unavailability can be expressed as

$$U_{sys} \triangleq \frac{\overline{r_{sys}}}{\overline{r_{sys}} + \overline{m_{sys}}}$$

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# Reliability of repairable components in parallel

- The *average time to failure* (or *operation time*, or *uptime*) of the parallel system can be expressed as:

$$\overline{m_{sys}} = \frac{1 + \lambda_1 \overline{r_1} + \lambda_2 \overline{r_2}}{\lambda_1 \lambda_2 (\overline{r_1} + \overline{r_2})}$$

- The failure rate of the parallel system is $\overline{\lambda_{sys}} \triangleq \frac{1}{\overline{m_{sys}}}$

$$\overline{\lambda_{sys}} = \frac{\lambda_1 \lambda_2 (\overline{r_1} + \overline{r_2})}{1 + \lambda_1 \overline{r_1} + \lambda_2 \overline{r_2}}$$

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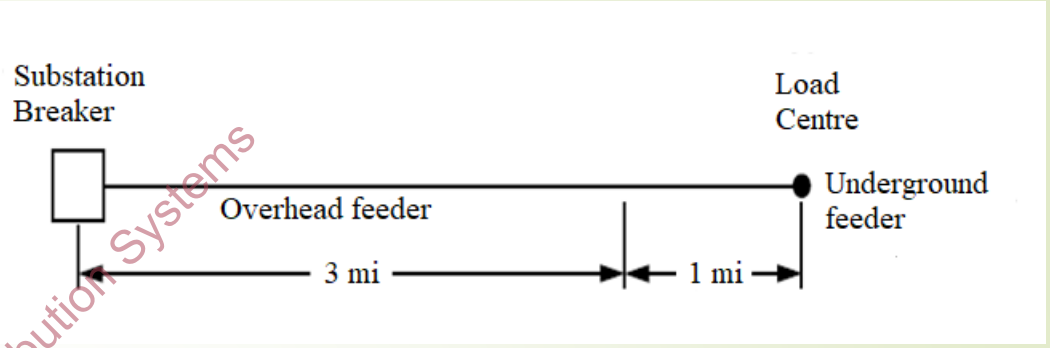
# Example…




- The Figure shows a 4 mile long distribution feeder. Approximately 1 mile of the feeder has been built underground, while the rest of the feeder is overhead. The underground feeder has two termination points. Two faults per circuit-mile for the overhead section and one fault per circuit-mile for the underground section of the feeder have been recorded in the last 10 years. The annual cable termination fault rate is given as 0.5% per cable termination. Furthermore, based on past experience, it is known that the average repair times for the overhead section, underground section, and each cable termination are 3, 30, and 3 h, respectively.

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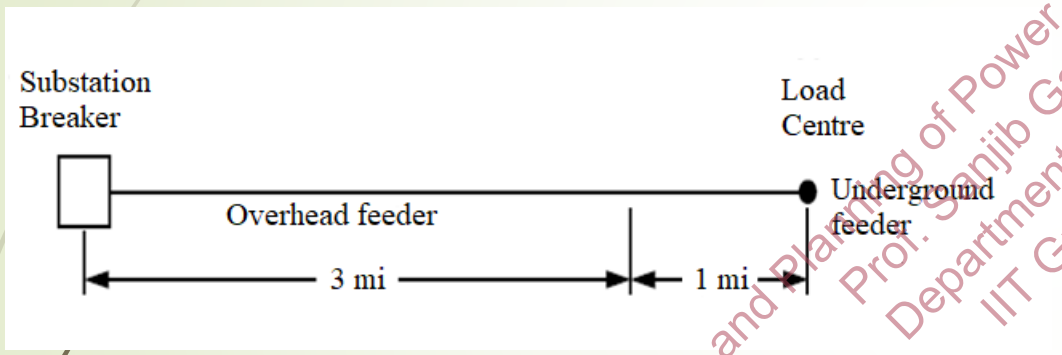
Ref.: T. Gonen. *Electric Power Distribution System Engineering*

# Example…

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Using the given information, determine the following:

- a. Total annual fault rate of the feeder
- b. Average annual fault restoration time of the feeder in hours
- c. Unavailability of the feeder
- d. Availability of the feeder

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# Solution…

- a. Total annual fault rate of the feeder is

$$\lambda_{FDR}=\sum_{i=1}^{3}\lambda_i = \lambda_{OH}+\lambda_{UG}+2\lambda_{CT}$$

Where
$\lambda_{OH}$ is the total annual fault rate of overhead section of feeder
$\lambda_{UG}$ is the total annual fault rate of underground section of feeder
$\lambda_{CT}$ is the total annual fault rate of cable terminations

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$$\lambda_{FDR}=3\left(\frac{2}{10}\right)+1\left(\frac{1}{10}\right)+2(0.005)$$

=0.71 faults/year

# Solution…

- b. Average fault restoration time of the feeder per fault is:

$$\overline{r_{FDR}} = \sum_{i=1}^{3} \overline{r_i} = \overline{r_{OH}} + \overline{r_{UG}} + 2\overline{r_{CT}}$$

Where
$\overline{r_{OH}}$ is the average repair time for overhead section of feeder, h
$\overline{r_{UG}}$ is the average repair time for underground section of feeder, h
$\overline{r_{CT}}$ is the average repair time per cable termination, h

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$$\overline{r_{FDR}} = 3+30+2(3)$$
$$= 39 \text{ hr}$$

# Solution…

- However, the average annual fault restoration time of the feeder is:

$$\bar{r}_{FDR} = \frac{\sum_{i=1}^{3} \lambda_i \bar{r}_i}{\sum_{i=1}^{3} \lambda_i}$$

$$\bar{r}_{FDR} = \frac{(l_{OH}\lambda_{OH})(\overline{r_{OH}}) + (l_{UG}\lambda_{UG})(\overline{r_{UG}}) + (2\lambda_{CT})(\overline{r_{CT}})}{\lambda_{FDR}}$$

$$= \frac{(3\times0.2)(3)+(1\times0.1)(30)+(2\times0.005)(3)}{0.71} = \frac{4.83}{0.71}$$

=6.8 hr

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# Solution…

- Unavailability of the feeder is:

$$U_{FDR} = \frac{\bar{r}_{FDR}}{\bar{r}_{FDR} + \bar{m}_{FDR}}$$

$$U_{FDR} = \frac{6.8}{6.8 + 12338}$$

$$= 0.055\%$$

$$\overline{m_{FDR}} = \frac{1}{\lambda_{FDR}} = 8760 * \frac{1}{0.71} = 12338$$

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# Solution…

- d. Availability of the feeder in percentage is:

$$A_{FDR} = 100 - U_{FDR}$$

$$=100-0.055$$

$$=99.945\%$$

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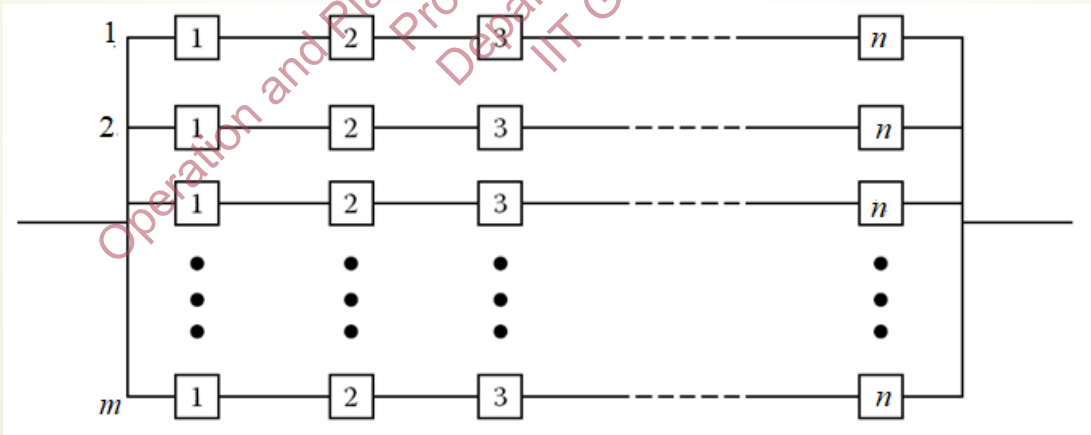
# Reliability for the Parallel-Series Combination

- Simple combinations of series and parallel subsystems (or components) can be analyzed by successively reducing subsystems into equivalent parallel or series components.

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# Reliability for the Parallel-Series Combination

- The equivalent reliability of the system with *m* parallel paths of *n* components each can be expressed as:

$$R_{sys}=1-(1-R^n)^m$$

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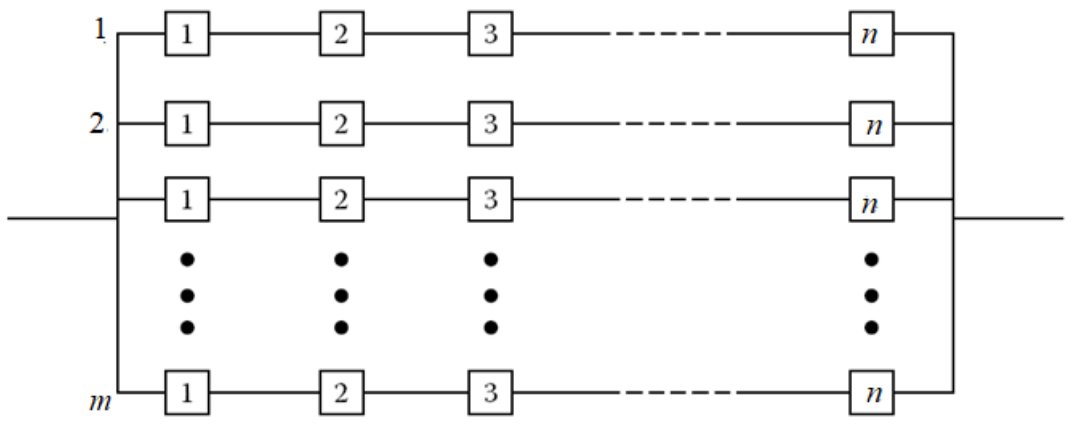
Where
$R_{sys}$ is the equivalent reliability of system
$R^n$ is the equivalent reliability of a path
$R$ is the reliability of a component
$n$ is the total number of components in a path
$m$ is the total number of paths

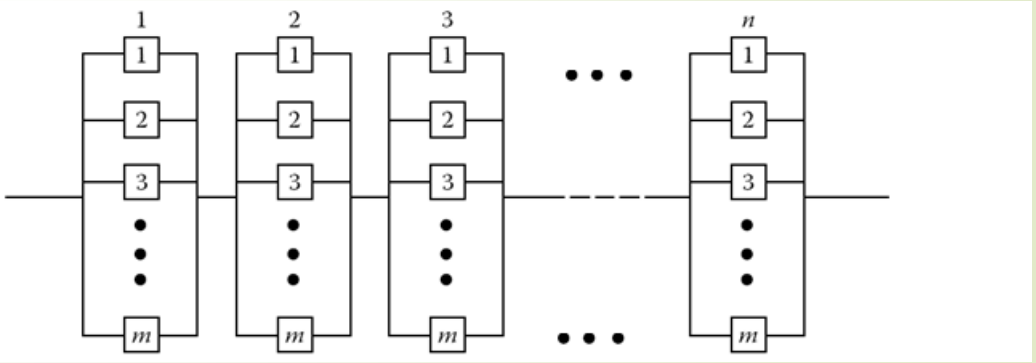


# Reliability for the Series-Parallel Combination

- The equivalent reliability of the system of *n* series units (or banks) with *m* parallel components in each unit (or bank) can be expressed as:

$$R_{sys}=[1-(1-R)^m]^n$$

Where
$R_{sys}$ is the equivalent reliability of system
$1-(1-R)^m$ is the equivalent reliability of a parallel unit (or bank)
$R$ is the reliability of a component
$m$ is the total number of components in a parallel unit (or bank)
$n$ is the total number of units (or banks)

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# Example…

- Consider the various combinations of the reliability block diagrams shown in Figure. Assume that they are based on the logic diagrams of each subsystem and that the reliability of each component is 0.85. Determine the equivalent system reliability of each configuration.

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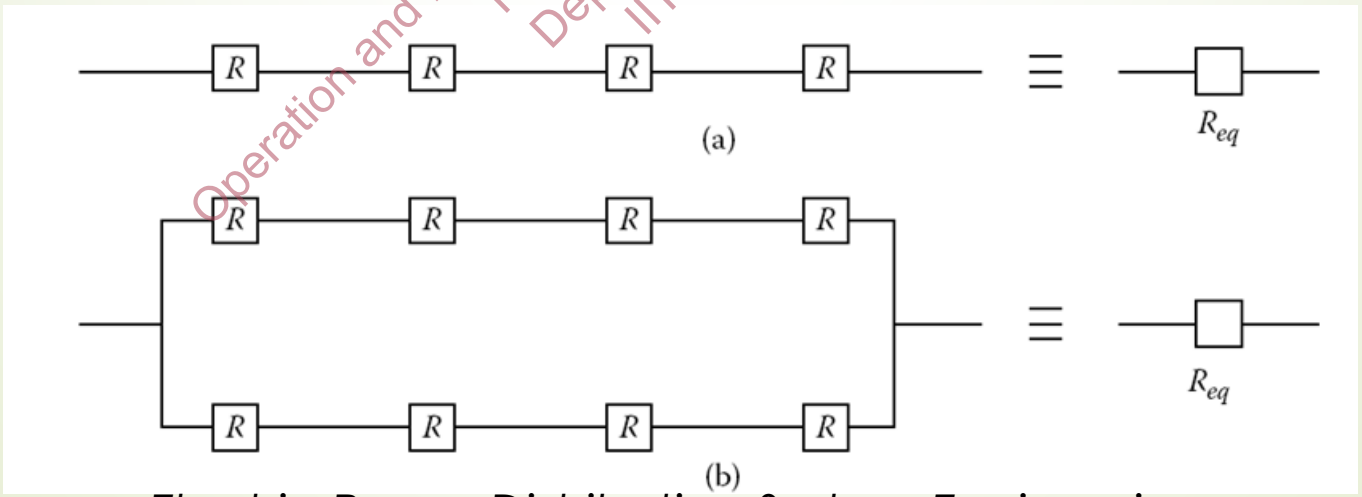
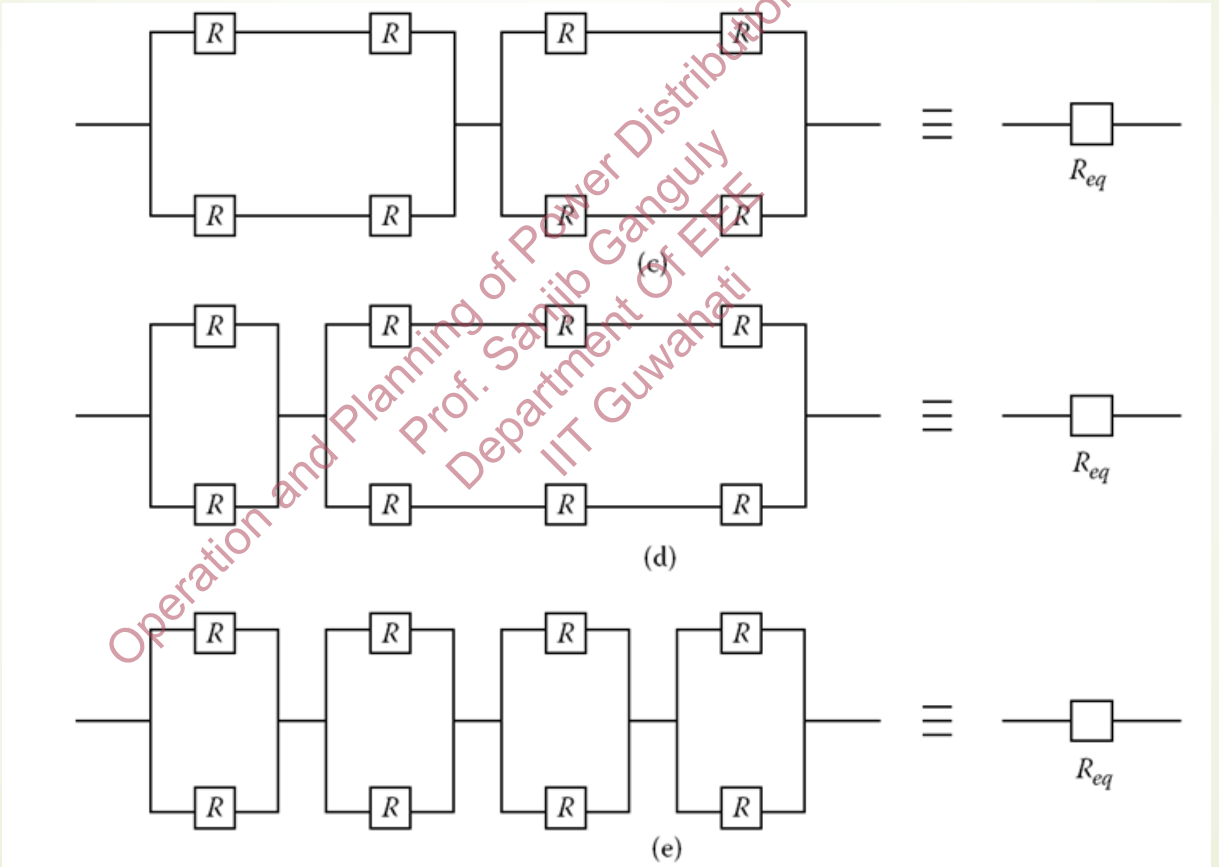



Ref.: T. Gonen. *Electric Power Distribution System Engineering*

# Example…

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# Solution…

- (a) The equivalent system reliability for the series system is:

$$R_{eq} = R_{sys} = \prod_{i=1}^{4} R_i = (0.85)^4 = 0.5220$$

- (b) The equivalent system reliability for the parallel–series system is: $R_{eq} = 1 - (1 - R^4)^2 = 1 - [1 - (0.85)^4]^2$
  $= 0.7715$

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# Solution…

- (c) The equivalent system reliability for the mixed-parallel system is:

$$R_{eq} = [1 - (1 - R^2)^2 \, [1 - (1 - R^2)^2]$$
$$= [1 - (1 - 0.85^2)^2 \, [1 - (1 - 0.85^2)^2]$$
$$= 0.8519$$

- (d) The equivalent system reliability for the mixed-parallel system is:

$$R_{eq} = [1 - (1 - R)^2 \, [1 - (1 - R^3)^2]$$
$$= [1 - (1 - 0.85)^2 \, [1 - (1 - 0.85^3)^2]$$
$$= 0.8320$$

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# Solution…

- (e) The equivalent system reliability for the series–parallel system from Equation is:

$$R_{eq} = [1-(1-R)^2]^4$$

$$=[1-(1-0.85)^2]^4$$

$$= 0.9130$$

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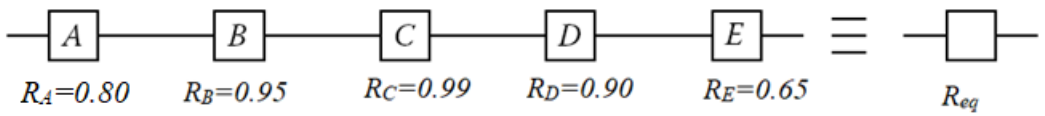
# Example…

- Assume that a system has five components, namely, *A*, *B*, *C*, *D*, and *E*, as shown in Figure, and that each component has different reliability as indicated in the figure. Determine the following:
- a. The equivalent system reliability.
- b. If the equivalent system reliability is desired to be at least 0.8, or 80%, design a system configuration to meet this system requirement by using each of the five components at least once.

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# Solution…

- (a) The equivalent system reliability is:

$$R_{eq} = \prod_{i=1}^{5} R_i$$

$$= (0.80)(0.95)(0.99)(0.90)(0.65)$$

$$=0.4402 \quad \text{or } 44.02\%$$

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# Solution…

- (b) In general, the best way of improving the overall system reliability is to protect the less-reliable components with parallel components. Therefore, since the relatively less-reliable components are *A* and *E*, these can be backed by parallel redundancy as shown in Figure. Therefore, the new equivalent system reliability becomes

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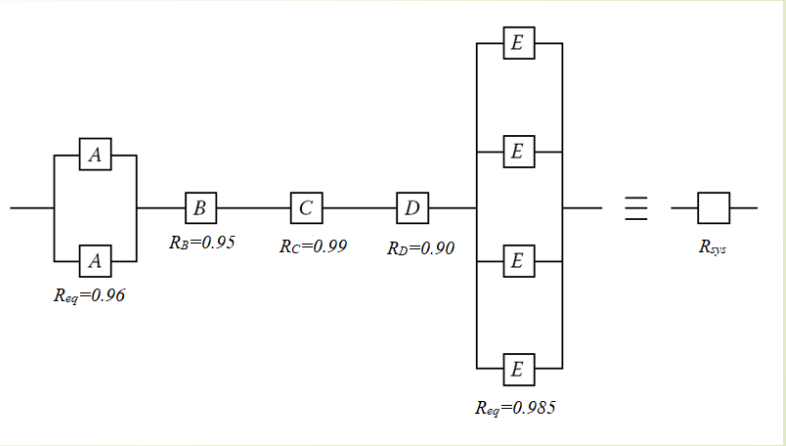
$$R_{sys} = \prod_{i=1}^{5} R_i$$

$$= [1 - (1 - 0.80)^2](0.95)(0.99)(0.90)[1 - (1 - 0.65)^4$$

$$= 0.8004 \ \ or \ 80.04\%$$



# Example…

- Assume that a three-phase transformer bank consists of three single-phase transformers identified as *A*, *B*, and *C* for the sake of convenience. Assume that (1) transformer *A* is an old unit and therefore has a reliability of 0.90, (2) transformer *B* has been in operation for the last 20 years and therefore has been estimated to have a reliability of 0.95, and (3) transformer *C* is a brand new one with a reliability of 0.99. Based on the given information and assumption of independence, determine the following:

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# Example…

- a. The probability of having no failing transformer at any given time.
- b. If one out of the three transformers fails at any given time, what are the probabilities for that unit being the transformer *A*, or *B*, or *C*?
- c. If two out of the three transformers fail at any given time, what are the probabilities for those units being the transformers *A* and *B*, or *B* and *C*, or *C* and *A*?
- d. What is the probability of having all three transformers out of service at any given time?

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# Solution…

- (a) The probability of having all transformers operational without fail at any given time is:

$$P[A \cap B \cap C] = P(A)P(B)P(C)$$

$$= (0.90)(0.95)(0.99)$$

$$= 0.84645$$

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# Solution…

- (b) If one out of the three transformers fails at any given time, the probabilities for that unit being the transformer *A*, or *B*, or *C* are:

$$P[\overline{A} \cap B \cap C] = P(\overline{A})P(B)P(C)$$
$$= (0.10)(0.95)(0.99)$$
$$= 0.09405$$

$$P[A \cap \overline{B} \cap C] = P(A)P(\overline{B})P(C)$$
$$= (0.90)(0.05)(0.99)$$
$$= 0.04455$$

$$P[A \cap B \cap \overline{C}] = P(A)P(B)P(\overline{C})$$
$$= (0.90)(0.95)(0.01)$$
$$= 0.00855$$

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# Solution…

- If two out of the three transformers fail at any given time, the probabilities for those units being the transformers *A* and *B*, or *B* and *C*, or *C* and *A* are

$$P[\overline{A} \cap \overline{B} \cap C] = P(\overline{A})P(\overline{B})P(C)$$
$$= (0.10)(0.05)(0.99)$$
$$= 0.00495$$

$$P[A \cap \overline{B} \cap \overline{C}] = P(A)P(\overline{B})P(\overline{C})$$
$$= (0.90)(0.05)(0.01)$$
$$= 0.00045$$

$$P[\overline{A} \cap B \cap \overline{C}] = P(\overline{A})P(B)P(\overline{C})$$
$$= (0.10)(0.95)(0.01)$$
$$= 0.00095$$

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# Solution…

- The probability of having all three transformers out of service at any given time is:

$$P\left[\overline{A} \cap \overline{B} \cap \overline{C}\right] = P\left(\overline{A}\right)P\left(\overline{B}\right)P\left(\overline{C}\right)$$

$$= (0.10)(0.05)(0.01)$$

$$= \ 0.00005$$

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# Summary of the solution…

Summary of the Computations

| Number of failed Transformers | System Modes | Probability |
|---|---|---|
| 0 | $A \cap B \cap C$ | 0.84645 |
| 1 | $\overline{A} \cap B \cap C$ | 0.09405 |
| | $A \cap \overline{B} \cap C$ | 0.04455 |
| | $\bar{A} \cap B \cap \overline{C}$ | 0.00855 |
| 2 | $\overline{A} \cap \overline{B} \cap C$ | 0.00495 |
| | $A \cap \overline{B} \cap \overline{C}$ | 0.00045 |
| | $\overline{A} \cap B \cap \overline{C}$ | 0.00095 |
| 3 | $\overline{A} \cap \overline{B} \cap \overline{C}$ | 0.00005 |

$$= \sum 1.00000$$

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# Reliability modelling: Markov process

- In reliability studies, the variable *t* represents time, and *X*(*t*) describes the state of the system at time *t*. The states at a given time *tn* actually represent the (exhaustive and mutually exclusive) outcomes of the system at that time.
- A *Markov process* is a stochastic system for which the occurrence of a future state depends on the immediately preceding state and only on it.
- The Markovian process is characterized by a lack of memory.

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# Reliability modelling: Markov process

- A discrete parameter stochastic process, $\{X(t); t = 0, 1, 2,\ldots\}$, or a continuous parameter stochastic process, $\{X(t); t \geq 0\}$, is a Markov process if it has the following *markovian property*:

$$P\{X(t_n) = x_n | X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_{n-1}) = x_{n-1}\} = P\{X(t_n) = x_n | X(t_{n-1}) = x_{n-1}\}$$

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# Reliability modelling: Markov process

- For any set of $n$ time points, $t_1 < t_2 < \ldots < t_n$ in the index set of the process, and any real numbers $x_1, x_2, \ldots, x_n$. The probability is called the *transition probability* and represents the *conditional probability* of the system being in $x_n$ at $t_n$, given that it was $x_n-1$ at $t_n-1$. It is also called the *one-step transition probability* due to the fact that it represents the system between $t_n-1$, and $t_n$.

$$P_{x_{n-1},x_n} = P\{x(t_n) = x_n \,|\, X(t_{n-1}) = x_{n-1}\}$$

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# Reliability modelling: Markov process

- For any set of *n* time points, $t_1 < t_2 < \ldots < t_n$ in the index set of the process, and any real numbers $x_1, x_2, \ldots, x_n$. The probability of *k*-step transition is,

$$P_{x_n,x_{n+k}} = P\{X(t_{n+k}) = x_{n+k} | X(t_n) = x_n\}$$

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# Reliability modelling: Markov process

- A *Markov chain* is defined by a sequence of discrete-valued random variables, $\{X(t_n)\}$, where $t_n$ is discrete-valued or continuous. Therefore, one can also define the Markov chain as the *Markov process with a discrete state space*.

$$p_{ij} = P\{X(t_n) = j | X(t_{n-1}) = i\}$$

- As the one-step transition probability of going from state $i$ at $t_n - 1$ to state $j$ at $t_n$ assumes that these probabilities do not change over time. The term used to describe this assumption is stationarity.

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# Reliability modelling: Markov process

- A Markov chain is completely defined by its transition probabilities, of going from state *i* to state *j*, given in a matrix form:

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} & \cdots & p_{0n} \\ p_{10} & p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{20} & p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ p_{30} & p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{n0} & p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{bmatrix}$$

$$\sum_{j}^{n} p_{ij} = 1 \; for \; all \; i$$

$$p_{ij} \geq 0 \; for \; all \; ij$$

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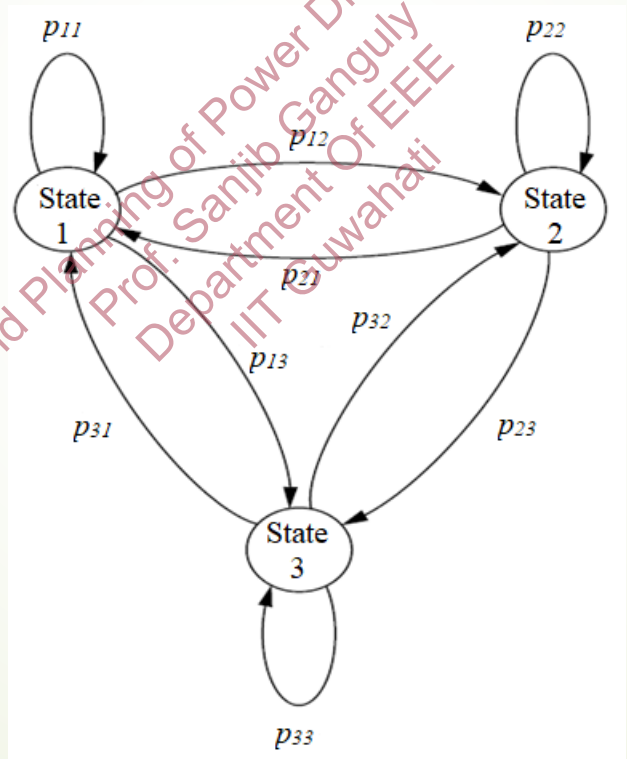
The matrix **P** is called a *one-step transition matrix* (or *stochastic matrix*), since all the transition probabilities $p_{ij}$'s are fixed and independent of time.

# Reliability modelling: Markov process

- Transition diagram:

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# Reliability modelling: Markov process

- Example: The records of distribution utility indicate that only 2% of the transformers that are presently down and therefore being repaired now will be down and therefore will need repair next time. The records also show that 5% of those transformers that are currently up and therefore in service now will be down and therefore will need repair next time. Assuming that the process is discrete, markovian, and has stationary transition probabilities, determine the transition diagram.

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Ref.: T. Gonen. *Electric Power Distribution System Engineering*

# Reliability modelling: Markov process

- Let *t* and *t* + 1 represent the present time (i.e., *now*) and the next time, respectively. Therefore, the associated conditional probabilities are:

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$$P\{X_{t+1} = down | X_t = down\} = 0.02$$

$$P\{X_{t+1} = up | X_t = down\} = 0.98$$

$$P\{X_{t+1} = down | X_t = up\} = 0.05$$

$$P\{X_{t+1} = up | X_t = up\} = 0.95$$

# Reliability modelling: Markov process

- Let numbers 1 and 2 represent the states of down and up, respectively.

$p_{11} = 0.02$ $p_{12} = 0.98$

$p_{21} = 0.05$ $p_{22} = 0.95$

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.02 & 0.98 \\ 0.05 & 0.95 \end{bmatrix}$$

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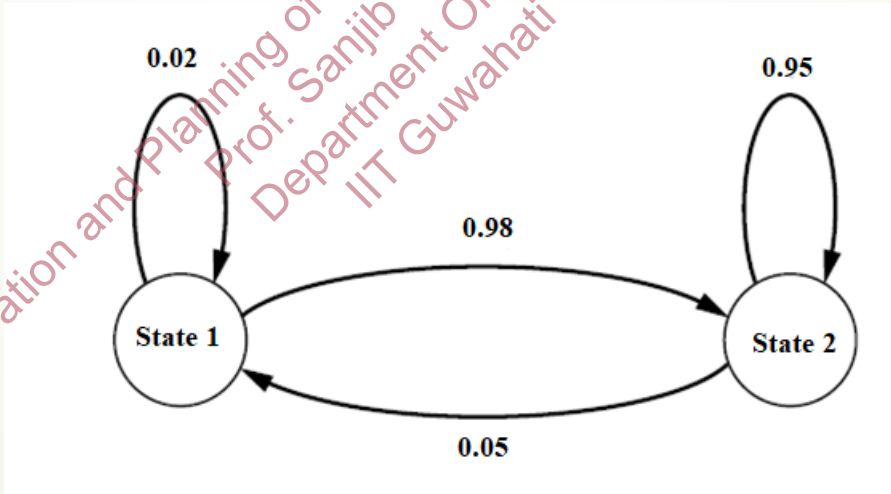
# Reliability modelling: Markov process

- State-transition diagram:

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# Reference


- T. Gonen. *Electric Power Distribution System Engineering*; CRC Press, 3rd Edition, 2014.


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# Thank You…

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